Feb. 22, 2017

Sect. 4-3

Solving Rational Equations and Inequalities

Solve: $\frac{6}{x^2} - \frac{5}{x} = \frac{1}{1}$ LCD: x^2

$x^2\left(\frac{6}{x^2}\right) - x^2\left(\frac{5}{x}\right) = x^2(1)$

$6 - 5x = x^2$

$x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$x = -6$ * ✓ $x = -1$ * ✓

* Must check. Make denom = 0?

$$\frac{x^2}{1} + \frac{x}{x-1} = \frac{x}{1} + \frac{x^3}{x-1} \qquad \text{LCD: } x-1$$

$$(x-1)(x^2) + (x-1)\left(\frac{x}{x-1}\right) = (x-1)(x) + (x-1)\left(\frac{x^3}{x-1}\right)$$

$$x^3 - x^2 + x = x^2 - x + x^3$$

$$-2x^2 + 2x = 0$$

$$-2x(x-1) = 0$$

$-2x = 0 \qquad x - 1 = 0$

$x = 0 \qquad x = 1$

So $\{0\}$

$$\frac{x-1}{x^2+3x+2} + \frac{2x}{x+2} = \frac{x-1}{x+1}$$

LCD: $(x+2)(x+1)$

$(x+2)(x+1)$

$$(x+2)(x+1)\left(\frac{x-1}{(x+2)(x+1)}\right) + (x+2)(x+1)\left(\frac{2x}{x+2}\right) = (x+2)(x+1)\left(\frac{x-1}{x+1}\right)$$

$$x-1 + 2x(x+1) = (x+2)(x-1)$$

$$x-1+2x(x+1)=(x+2)(x-1)$$

$$x-1+2x^2+2x=x^2+x-2$$

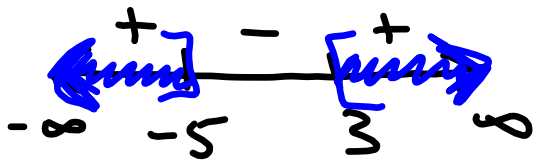
$$x^2+2x+1=0$$

$$(x+1)(x+1)=0$$

$$x=-1 \quad *$$

No Sol.

$$\frac{x+5}{x-3} \geq 0$$



$$(-\infty, -5] \cup [3, \infty)$$

$$\frac{x+5}{x-3} > 2$$

If $x > 3$

$x+5 > 2(x-3)$

$x+5 > 2x-6$

$11 > x$

$x < 11$

$3 < x < 11$

So

$(3, 11)$

If $x < 3$

$x+5 < 2(x-3)$

$x+5 < 2x-6$

$11 < x$

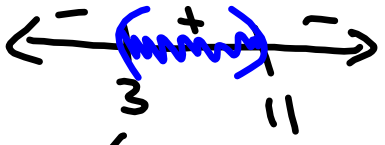
~~$x > 11$~~ ?

~~and $x < 3$~~

$$\frac{x+5}{x-3} > 2$$

$$\frac{x+5}{x-3} - \frac{2}{1} > 0 \Rightarrow \frac{x+5}{x-3} - \frac{2(x-3)}{x-3} > 0$$

$$\frac{x+5-2x+6}{x-3} > 0 \Rightarrow \frac{-x+11}{x-3} > 0$$



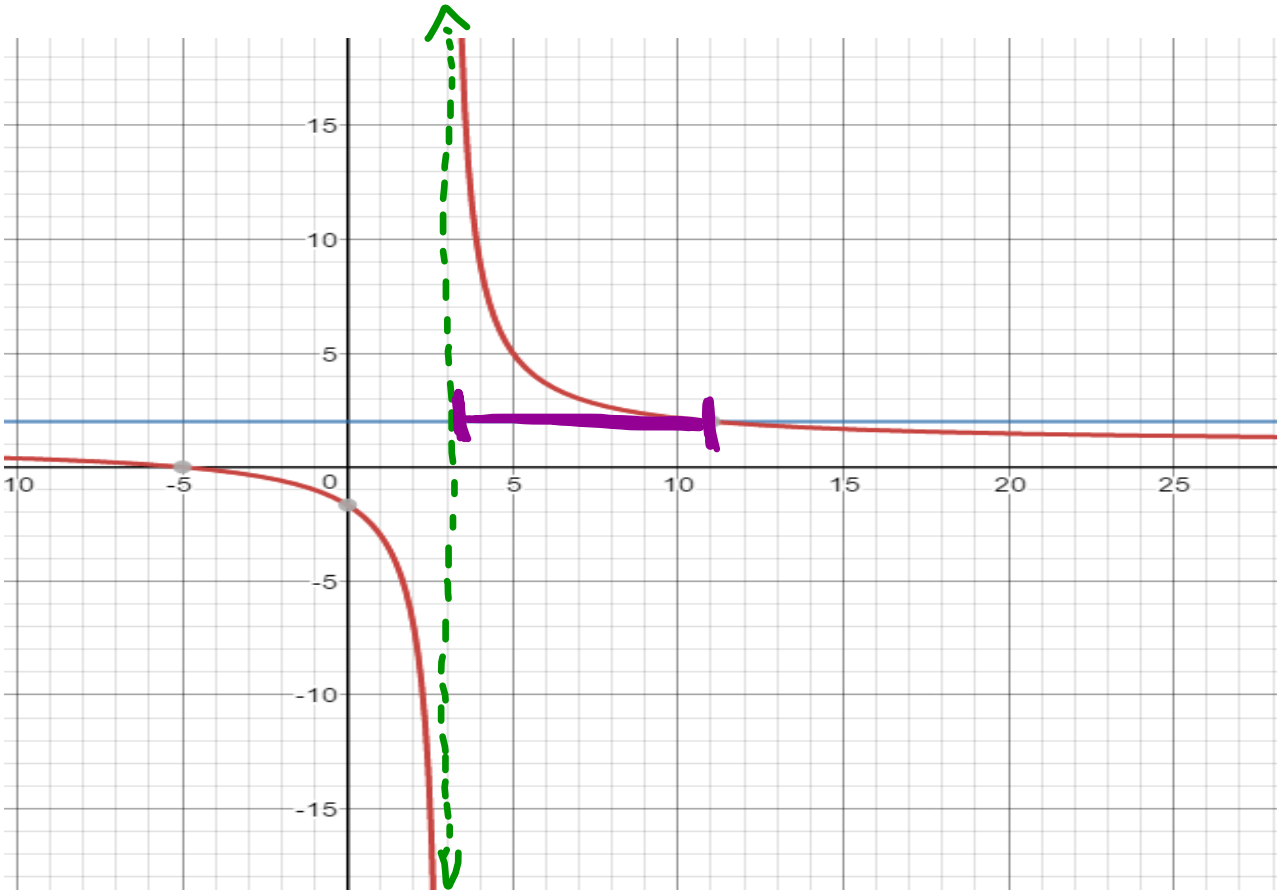
$(3, 11)$

$$\frac{x+5}{x-3} > 2$$

$y = \frac{x+5}{x-3}$

$y = 2$

$(3, 11)$



$$\frac{x^2-1}{x^2+x+3} \leq 0$$

$$[-1, 1]$$